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Reconstruction of $5D$ Cosmological Models From Recent Observations

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We use a parameterized equation of state (EOS) of dark energy to a $5D$ Ricci-flat cosmological solution and suppose the universe contains two major components: dark matter and dark energy. Using the recent observational datasets: the latest 182 type Ia Supernovae Gold data, the 3-year WMAP CMB shift parameter and the SDSS baryon acoustic peak, we obtain the best fit values of the EOS and two major components' evolution. We find that the best fit EOS crossing -1 in the near past $z \simeq 0.07$, the present best fit value of $w_x(0) < -1$ and for this model the universe experiences the acceleration at about $z \simeq 0.5$.

Keywords: Kaluza-Klein theory; cosmology; dark energy

1. Introduction

Observations of Cosmic Microwave Background (CMB) anisotropies¹, high redshift type Ia supernovae² and the surveys of clusters of galaxies³ indicate that an exotic component with negative pressure dubbed dark energy dominates the present universe. The most obvious candidate for this dark energy is the cosmological constant Λ with equation of state ($w_\Lambda = -1$), which is consistent with recent observations^{1,4} in 2σ region. However, it raises several theoretical difficulties^{5,6}. This has lead to models for dark energy which evolves with time, such as quintessence⁷, phantom⁸, quintom⁹, K-essence¹⁰, tachyonic matter¹¹ and so on. For this kind of models, one can design many kinds of potentials¹² and then study EOS for the dark energy. Another way is to use a parameterization of the EOS to fit the observational data, and then to reconstruct the potential and the evolution of the universe¹³. Various parameterization of the EOS of dark energy have been presented and investigated^{14,15,16,17}.

If the universe has more than four dimensions, general relativity should be extended from $4D$ to higher dimensions. One of such extensions is the $5D$ Space-Time-Matter (STM) theory^{18,19} in which our universe is a $4D$ hypersurface floating

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in a $5D$ Ricci-flat manifold. This theory is supported by Campbell's theorem which states that any analytical solution of the ND Einstein equations can be embedded in a $(N + 1)D$ Ricci-flat manifold²⁰. A class of cosmological solutions of the STM theory is given by Liu and Mashhoon²², the authors restudied the solutions and pointed out that it can describe a bounce universe. It was shown that dark energy models, similar as the $4D$ quintessence and phantom ones, can also be constructed in this $5D$ cosmological solution in which the scalar field is induced from the $5D$ vacuum^{23,24}. The purpose of this paper is to use a model-independent method to reconstruct a $5D$ cosmological model and then study the universe evolution and the EOS of the dark energy which is constrained by recent observational data: the latest observations of the 182 Gold SNe Ia²⁵, the 3-year WMAP CMB shift parameter^{4,26} and the SDSS baryon acoustic peak²⁷. The paper is organized as follows. In Section 2, we briefly introduce the $5D$ Ricci-flat cosmological solution and derive the densities for the two major components of the universe. In Section 3, we will reconstruct the evolution of the model from cosmological observations. Section 4 is a short discussion.

2. Dark energy in the $5D$ Model

The $5D$ cosmological model was described as before^{21,22,28,30}. In this paper we consider the case where the $4D$ induced matter $T^{\alpha\beta}$ is composed of two components: dark matter ρ_m and dark energy ρ_x , which are assumed to be noninteracting. So we have

$$\begin{aligned} \frac{3(\mu^2 + k)}{A^2} &= \rho_m + \rho_x, \\ \frac{2\mu\dot{\mu}}{A\dot{A}} + \frac{\mu^2 + k}{A^2} &= -p_m - p_x, \end{aligned} \quad (1)$$

with

$$\rho_m = \rho_{m0} A_0^3 A^{-3}, \quad p_m = 0, \quad (2)$$

$$p_x = w_x \rho_x. \quad (3)$$

From Eqs. (1) - (3) and for $k = 0$, we obtain the EOS of the dark energy

$$w_x = \frac{p_x}{\rho_x} = -\frac{2\mu\dot{\mu}/(A\dot{A}) + \mu^2/A^2}{3\mu^2 A^2 - \rho_{m0} A_0^3 A^{-3}}, \quad (4)$$

and the dimensionless density parameters

$$\Omega_m = \frac{\rho_m}{\rho_m + \rho_x} = \frac{\rho_{m0} A_0^3}{3\mu^2 A}, \quad (5)$$

$$\Omega_x = 1 - \Omega_m. \quad (6)$$

where ρ_{m0} is the current values of dark matter density.

Consider Eq. (4) where A is a function of t and y . However, on a given $y = \text{constant}$ hypersurface, A becomes $A = A(t)$, which means we consider a $4D$

supersurface embedded in a flat 5D spacetime. As noticed before^{29,30}, the term $\dot{\mu}/\dot{A}$ in (4) can now be rewritten as $d\mu/dA$. Furthermore, we use the relation

$$A_0/A = 1 + z, \quad (7)$$

as an ansatz^{29,30} and define $\mu_0^2/\mu^2 = f(z)$ (with $f(0) \equiv 1$), then we find that Eqs. (4)-(6) can be expressed in term of the redshift z as

$$w_x = -\frac{1 + (1 + z)d\ln f(z)/dz}{3(1 - \Omega_m)}, \quad (8)$$

$$\Omega_m = \Omega_{m0}(1 + z)f(z), \quad (9)$$

$$\Omega_x = 1 - \Omega_m, \quad (10)$$

$$q = -\frac{1 + z}{2}d\ln f(z)/dz. \quad (11)$$

where q is the deceleration parameter and $q < 0$ means our universe is accelerating. Now we conclude that if the function w_x is given, the evolution of all the cosmic observable parameters in Eqs. (8) - (11) could be determined uniquely. Then we adopt the parametrization of EOS as follows^{15,31}

$$w_x(z) = w_0 + w_1 \frac{z}{1 + z} \quad (12)$$

From Eq. (8) and Eq. (12), we can obtain the function $f(z)$

$$f(z) = \frac{1}{(1 + z) \left[\Omega_{m0} + (1 - \Omega_{m0})(1 + z)^{3w_0 + 3w_1} \exp\left(-\frac{3w_1 z}{1 + z}\right) \right]}. \quad (13)$$

In the next section, we will use the recent observational data to find the best fit parameter (w_0, w_1, Ω_{m0}).

3. The best fit parameters from cosmological observations

In a flat universe with Eq. (12), the Friedmann equation can be expressed as

$$H^2(z) = H_0^2 E(z)^2 = H_0^2 [\Omega_{m0}(1 + z)^3 + (1 - \Omega_{m0})(1 + z)^{3(1 + w_0 + w_1)} e^{\frac{-3w_1 z}{1 + z}}] \quad (14)$$

Then the knowledge of Ω_{m0} and $H(z)$ is sufficient to determine $w_x(z)$ with $H_0 = 72 \text{ km} \cdot \text{s}^{-1} \cdot \text{Mpc}^{-1}$ ³². We use the maximum likelihood method³³ to constrain the parameters.

The Gold dataset compiled by Riess et. al is a set of supernova data from various sources and contains 182 gold points by discarding all SNe Ia with $z < 0.0233$ and all SNe Ia with quality='Silver' from previously published data with 21 new points with $z > 1$ discovered recently by the Hubble Space Telescope²⁵. Theoretical model parameters are determined by minimizing the quantity

$$\chi_{SNe}^2(\Omega_{m0}, w_0, w_1) = \sum_{i=1}^N \frac{(\mu_{obs}(z_i) - \mu_{th}(z_i))^2}{\sigma_{(obs;i)}^2} \quad (15)$$

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where $N = 182$ for Gold SNe Ia data, $\sigma_{(obs;i)}^2$ are the errors due to flux uncertainties, intrinsic dispersion of SNe Ia absolute magnitude and peculiar velocity dispersion respectively. These errors are assumed to be gaussian and uncorrelated. The theoretical distance modulus is defined as

$$\begin{aligned}\mu_{th}(z_i) &\equiv m_{th}(z_i) - M \\ &= 5 \log_{10}(D_L(z)) + 5 \log_{10}\left(\frac{H_0^{-1}}{Mpc}\right) + 25\end{aligned}\quad (16)$$

where

$$D_L(z) = H_0 d_L(z) = (1+z) \int_0^z \frac{H_0 dz'}{H(z'; \Omega_{m0}, w_0, w_1)} \quad (17)$$

and μ_{obs} is given by supernovae dataset.

The shift parameter is defined as³⁴

$$\bar{R} = \frac{l_1'^{TT}}{l_1^{TT}} = \frac{r_s}{r_s'} \frac{d_A'(z_{rec})}{d_A(z_{rec})} = \frac{2}{\Omega_{m0}^{1/2}} \frac{q(\Omega_r', a_{rec})}{\int_0^z \frac{H_0 dz'}{H(z')}} \quad (18)$$

where z_{rec} is the redshift of recombination, r_s is the sound horizon, $d_A(z_{rec})$ is the sound horizon angular diameter distance, $q(\Omega_r', a_{rec})$ is the correction factor. For weak dependence of $q(\Omega_r', a_{rec})$, the shift parameter is usually expressed as

$$R = \Omega_{m0}^{1/2} \int_0^z \frac{H_0 dz'}{H(z'; \Omega_{m0}, w_0, w_1)} \quad (19)$$

The R obtained from 3-year WMAP data^{4,26} is

$$R = 1.70 \pm 0.03 \quad (20)$$

With the measurement of the R , we obtain the χ_{CMB}^2 expressed as

$$\chi_{CMB}^2(\Omega_{m0}, w_0, w_1) = \frac{(R(\Omega_{m0}, w_0, w_1) - 1.70)^2}{0.03^2} \quad (21)$$

The size of Baryon Acoustic Oscillation (BAO) is found by Eisenstein et al²⁷ by using a large spectroscopic sample of luminous red galaxy from SDSS and obtained a parameter A which does not depend on dark energy directly models and can be expressed as

$$A = \Omega_{m0}^{1/2} E(z_{BAO})^{-1/3} \left[\frac{1}{z_{BAO}} \int_0^z \frac{dz'}{E(z'; \Omega_{m0}, w_0, w_1)} \right]^{2/3} \quad (22)$$

where $z_{BAO} = 0.35$ and $A = 0.469 \pm 0.017$. We can minimize the χ_{BAO}^2 defined as³⁵

$$\chi_{BAO}^2(\Omega_{m0}, w_0, w_1) = \frac{(A(\Omega_{m0}, w_0, w_1) - 0.469)^2}{0.017^2} \quad (23)$$

To break the degeneracy of the observational data and find the best fit parameters, we combine these datasets to minimize the total likelihood χ_{total}^2 ³⁶

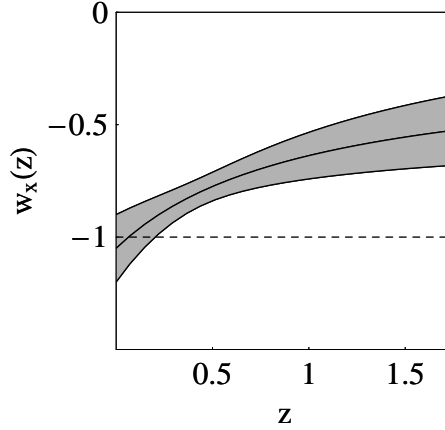


Fig. 1. The best fits of $w_x(z)$ with 1σ errors (shaded region).

$$\chi_{total}^2 = \chi_{SNe}^2 + \chi_{CMB}^2 + \chi_{BAO}^2 \quad (24)$$

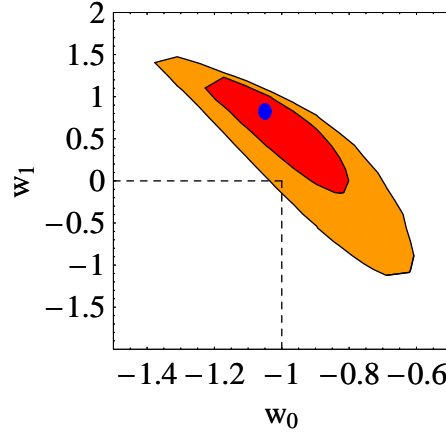
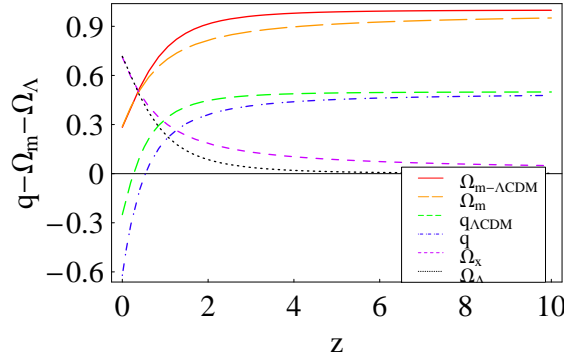
We obtain the best fit values (Ω_{m0}, w_0, w_1) are (0.288, -1.050, 0.824) and to identify the dependence of the best fit values of the parameters, we set Ω_{m0} to be fixed when calculating the confidence level of (w_0, w_1) . The errors of the best fit $w_x(z)$ are calculated using the covariance matrix³⁷ and shown in Fig.1. The corresponding χ^2 contours in parameters space (w_0, w_1) is shown in Fig.2.

From Fig.1 we find that $w_x(z)$ is constrained in a narrow space, the best fit $w_x(z)$ crosses -1 at about $z = 0.07$ and at present the best value of $w_x(0) < -1$, but in 1σ confidence level we can't rule out the possibility $w_x(0) > -1$. Fig.2 shows that a cosmological constant is ruled out in 1σ confidence level.

Using the function $f(z)$, the best fit values (Ω_{m0}, w_0, w_1) , we obtain Ω_m, Ω_x , the deceleration parameter q from Eq.(9)-(11) and their evolution is plotted in Fig.3. Fig.3 also shows the evolution of $q_{\Lambda CDM}, \Omega_{m-\Lambda CDM}, \Omega_{\Lambda}$ in a 4D flat ΛCDM model with the present $\Omega_{m0-\Lambda CDM} = 0.283$ obtained from above cosmological observations. We can see that the transition point from decelerated expansion to accelerated expansion with $q = 0$ is at $z \simeq 0.5$ and it is earlier than the ΛCDM model. Our universe experiences a expansion at present in a 4D supersurface embedded in a 5D Ricci-flat spacetime or in ΛCDM model.

4. Discussion

Observations indicate that our universe now is dominated by two dark components: dark energy and dark matter. The 5D cosmological solution presented by Liu, Mashhoon and Wesson in²¹ and²² contains two arbitrary functions $\mu(t)$ and $\nu(t)$, one of the two functions, $\mu(t)$, plays a similar role as the potential $V(\phi)$ in the

6 *Chengwu Zhang, Lixin Xu, Yongli Ping and Hongya Liu*Fig. 2. The contours show 2-D marginalized 1σ and 2σ confidence limits in the (w_0, w_1) planeFig. 3. The evolution of $q(z)$, Ω_m , Ω_x and $q_{\Lambda CDM}$, $\Omega_{m-\Lambda CDM}$, Ω_Λ from 5D cosmological model and ΛCDM model respectively.

quintessence and phantom dark energy models, which can easily change to another arbitrary function $f(z)$. Thus, if the current values of the matter density parameter Ω_{m0} , w_0 and w_1 in the EOS are all known, this $f(z)$ could be determined uniquely. In this paper we mainly focus on the constraints on this model from recent observational data: the 182 Gold SNe Ia, the 3-year WMAP CMB shift parameter and the SDSS baryon acoustic peak. Our results show that the recent observations allow for a narrow variation of the dark energy EOS and the best fit dynamical $w_x(z)$ crosses -1 in the recent past. Using the best fit values (Ω_{m0}, w_0, w_1) , we have studied the evolution of the dark matter density Ω_m , the dark energy density Ω_x and the deceleration parameter q in a $4D$ supersurface of $5D$ spacetime, which is similar to the ΛCDM model. In the future, we hope that more and precision cosmological

observations could determine the key points of the evolution of our universe, such as the transition point from deceleration to acceleration, then distinguish the 5D cosmological model from others.

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